Magnetostatic surface waves on left-handed materials (LHM)

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ABSTRACT

The nonlinear characteristics of magnetostatic surface waves at microwave frequencies in a layered structure of left-handed material film and a semi-infinite linear ferrite substrate have been investigated. The general dispersion relation is derived and analyzed numerically. It is found that it has two solutions for ω (k),one represents a physical solution and other unacceptable. The effects of the applied external magnetic field around the proposed region have also been examined.

Keywords: Left-handed materials; Dispersion relation; Magnetostatic surface waves; wave-guides; Boundary method.

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INTRODUCTION:

Left-handed material (LHM)[1-7] have been receiving an increasing interest because of their proposed applications in future microwave engineering technology. Recently, Shelby et al[1] have demonstrated a two dimensionally isotropic left-handed material which consist of a two dimensionally periodic array of copper split ring resonators and wires (SSR). The LHM are at certain band of frequencies, behaves with negative $\varepsilon(\omega)$ and $\mu(\omega)$, and also have imaginary part, thereby the refractive index of such metamaterial exhibits a negative value in this frequency as produced by Veslago[2]. A LHM also verify some of the explicit prediction such as reversed refraction, backward, Cherenkove radiation and reversed Doppler effect[3]. The study of nonlinear optical effects in the various waveguides structures containing a gyromagnetic media such as a ferrite[8] is also considered a key problem of the simulation of a number of opto-microwave electronic devices.

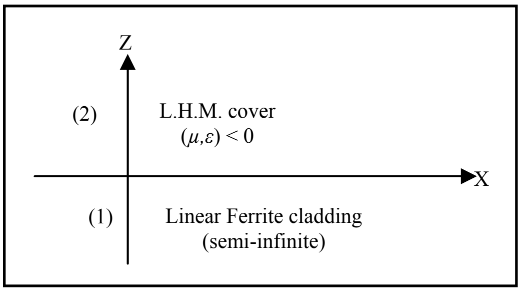

Fig. 1. Coordinate system for the single interface between LHM and a linear ferrite cladding, the applied magnetic field is in the Y-direction.

In this paper, we investigate the propagation characteristics of nonlinear magnetostatic surface waves at microwave frequencies in a layered structure of left-handed metamaterial cover and a linear ferrite substrate as shown in fig. 1.

Basic Equations:

The guiding structure to be considered consist of a LHM film which characterized by[6]:

$$\mu_{eff}(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \quad , \quad \varepsilon_{eff}(\omega) = 1 - \frac{\omega_{ep}^2}{\omega^2} \qquad (1)$$

Where $F = 0.56$, $\omega_0 / 2\pi = 4$ GHz , and $\omega_p / 2\pi = 10$ GHz

And, a gyromagnetic ferrite (YIG) substrate is described by a magnetic permeability tensor as:

$$\mu(\omega) = \begin{pmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_B & 0 \\ -\mu_{xz} & 0 & \mu_{xx} \end{pmatrix} \qquad (2)$$

Where,

$$\mu_{xx} = \mu_B \left(\frac{\omega_0(\omega_0 + \omega_m) - \omega^2}{\omega_0^2 - \omega^2} \right), \quad \mu_{xz} = i\mu_B \frac{\omega\omega_m}{\omega_0^2 - \omega^2} \qquad (3)$$

and μ_B is the usual Polder tensor elements, ω is the angular frequency of the supported wave, $\omega_0 = \gamma\mu_0 H_0$, $\omega_m = \gamma\mu_0 M_0$, H_0 is the applied magnetic field, $\gamma = 1.76\times10^{11}$ $S^{-1}T^{-1}$ is the gyromagnetic ratio, M_0 is the dc saturation magnetization of the magnetic insulator and μ_B has been introduced as the background, optical magnon permeability.

The electric and the magnetic field of TE wave propagating in the x-direction can be written as:

$$E = (0, E_y, 0)\exp[ik_0(\beta z - ct)] \qquad (4)$$

$$H = (H_x, 0, H_z)\exp[ik_0(\beta z - ct)] \qquad (5)$$

Where $\beta = \frac{k}{k_0}$ is the complex effective wave index constant, k_0 is the wave number of the free space, and c is the velocity of light in free space.

In the ferrite substrate:

The magnetostatic potential Ψ of the magnetostatic surface waves in the YIG film is written [8] as:

$$\Psi^{(1)} = A\exp(kx)e^{i(kx-\omega t)} \qquad (6)$$

The relevant component of the magnetic fields for the TE magnetostatic waves in the YIG can be written after considering the phase difference as:

$$h_x^{(1)} = ikA\exp(kz)e^{i(kx-\omega t)} \qquad (7)$$

$$h_z^{(1)} = -ikA\exp(kz)e^{i(kx-\omega t)} \qquad (8)$$

$$e_y^{(1)} = \frac{\omega\mu_0}{k}\left(-S\mu_{xz}h_x^{(1)} + \mu_{xx}h_z^{(1)}\right) \qquad (9)$$

Where, $S = \pm 1$, S = 1 stands for the propagation of the waves in forward direction, and S = -1 for the backward direction.

In LHM-cover:

Maxwell's Equations are:

$$\nabla \times E = i\omega\mu_0\mu_{eff}(\omega)H \qquad (10)$$

$$\nabla \times H = i\omega\varepsilon_0\varepsilon_{eff}(\omega)E \qquad (11)$$

Where the effective permeability and the effective permittivity both are less than zero.
From Eq. (10) we get:

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ ik & 0 & \dfrac{\partial}{\partial z} \\ 0 & E_y & 0 \end{pmatrix} = i\omega\mu_0\mu_{eff}(\omega) \begin{pmatrix} H_x \\ 0 \\ H_z \end{pmatrix} \qquad (12)$$

The components of the electric field and magnetic field are:

$$\frac{-\partial E_y}{\partial z} = i\omega\mu_0\mu_{eff}H_x \qquad (13)$$

Then we get:

$$H_x = \frac{i}{\omega\mu_0\mu_{eff}}\frac{\partial E_y}{\partial z} \qquad (13a)$$

Similarly,

$$ikE_y = i\omega\mu_0\mu_{eff}H_z \qquad (14)$$

Then we get:

$$H_z = \frac{k}{\omega\mu_0\mu_{eff}}E_y \qquad (14a)$$

Applying Eq. (11) the components of magnetic field is:

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ ik & 0 & \dfrac{\partial}{\partial z} \\ H_x & 0 & H_z \end{pmatrix} = -i\omega\varepsilon_0\varepsilon_{eff}(\omega) \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \qquad (15)$$

Then we get:

$$-ikH_z + \frac{\partial H_z}{\partial z} = -i\omega\varepsilon_0\varepsilon_{eff}E_y \qquad (16)$$

Substitute both Eq's. (13a) and (14a) in Eq. (16) respectively, we obtain:

$$-ik\left(\frac{k}{\omega\mu_0\mu_{eff}}\right)E_y + \frac{\partial}{\partial z}\left[\left(\frac{i}{\omega\mu_0\mu_{eff}}\right)\frac{\partial E_y}{\partial z}\right] = -i\omega\varepsilon_0\varepsilon_{eff}E_y \qquad (17)$$

Multiplying Eq. (17) by $\omega\mu_0\mu_{eff}$, we get:

$$\frac{\partial^2 E_y}{\partial z^2} - \left(k^2 - \omega^2\mu_0\varepsilon_0\mu_{eff}\varepsilon_{eff}\right)E_y = 0 \qquad (18)$$

$$\text{But } k_0^2 = \frac{\omega^2}{c^2} \quad \text{where, } \frac{1}{c^2} = \varepsilon_0 \mu_0 \tag{18a}$$

And $k = k_0\beta$ (18b)

Substitute both Eq's. (18a) and (18b) in Eq. (18), we obtain:

$$\frac{\partial^2 E_y}{\partial z^2} - \left(k_0^2 \beta^2 - k_0^2 \mu_{eff} \varepsilon_{eff}\right) E_y = 0 \tag{19}$$

$$\text{Let } k_1^2 = k_0^2 \left(\beta^2 - \mu_{eff} \varepsilon_{eff}\right) \tag{19a}$$

Finally, we got on a 2nd diff. eq. on the form:

$$\frac{\partial^2 E_y}{\partial z^2} - k_1^2 E_y = 0 \tag{20}$$

And its dolution as:

$$E_y = Ae^{k_1 z} \quad \text{and} \quad k_1 = k_0 \sqrt{\beta^2 - \mu_{eff} \varepsilon_{eff}} \tag{20a}$$

Where, A is a constant

The relevant components of magnetic fields and the electric field in LHM have the form:

$$H_x^{(2)} = \frac{iBk_1}{\omega \mu_0 \mu_{eff}} e^{k_1} e^{i(k_1 x - \omega t)} \tag{21}$$

$$H_z^{(2)} = \frac{Ak_1}{\omega \mu_0 \mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{22}$$

$$E_y^2 = Be^{k_1 z} e^{i(k_1 x - \omega t)} \tag{23}$$

$$\text{With } k_1 = k_0 \sqrt{\beta^2 - \mu_{eff} \varepsilon_{eff}} \tag{23a}$$

But for TE-waves it can be shown that[8] there is a $\frac{\pi}{2}$ phase difference between H_x and H_z. It is converted to redefine the field components as: $h_x = h_x$, $h_z = ih_z$ and $e_y = ie_y$, so the field components can be written in the left handed material cover as:

$$H_x^{(2)} = \frac{iBk_1}{\omega \mu_0 \mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{24}$$

$$H_z^{(2)} = \frac{iBk_1}{\omega \mu_0 \mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{25}$$

$$E_y = iBe^{k_1 z} e^{i(k_1 x - \omega t)} \tag{26}$$

Applying the boundary conditions for the continuity of tangential H at $z = 0$ we get:

$h_x^{(1)} = H_x^{(2)}$, then we have:

$$kAe^{i(kx-\omega t)} = \frac{Bk_1}{\omega\mu_0\mu_{eff}} e^{i(k_1 x-\omega t)} \qquad (27)$$

And, $e_y^{(1)} = E_y^{(2)}$, then we have:

$$-\omega\mu_0 A(S\mu_{xz} + \mu_{xx})e^{i(kx-\omega t)} = Be^{i(k_1 x-\omega t)} \qquad (28)$$

Dividing Eq. (27) by Eq. (28) we obtain:

$$\frac{k}{k_1} = \frac{-1}{\mu_{eff}}(S\mu_{xz} + \mu_{xx}) \qquad (29)$$

With, $k_1 = k\sqrt{\beta^2 - \varepsilon_{eff}\mu_{eff}}$ and $k = k_0\,\beta$ (29a)

Substitute Eq. (29a) in Eq. (29) and simplifying to get on the general dispersion equation.

$$\beta^2 = \frac{\varepsilon_{eff}\mu_{eff}(S\mu_{xz} + \mu_{xx})\,(S\mu_{xz} + \mu_{xx})}{(S\mu_{xz} + \mu_{xx})\,(S\mu_{xz} + \mu_{xx}) - \mu_{eff}^2} \qquad (30)$$

Numerical Results and discussions:

Numerical computations is performed in order to calculate the propagation characteristics of the nonlinear dispersion equation. The numerical calculations were carried out with the same parameters for the ferrite (YIG) substrate as[8]. the data parameters for the LHM as[5]: $F = 0.56$, $\omega_0 / 2\pi = 4$ GHz , and $\omega_p / 2\pi = 10$ GHz.

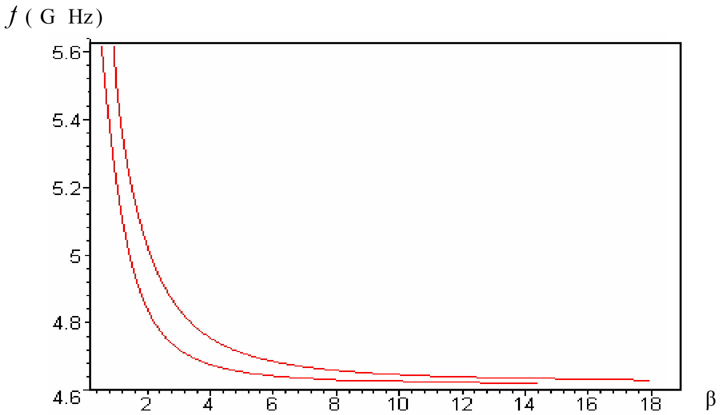

Fig. 2. Dispersion curves in both forward, backward direction, $\mu_0 H_0$ = 0.5 T, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ = 1.76 ×10^{11} rad s^{-1} T^{-1}, $\frac{\omega_p}{2\pi} = 10 \ \ GHz$, $\frac{\omega_0}{2\pi} = 4 \ \ GHz$, F = 0.56.

In this case, the frequency range in which both ε(ω) and μ(ω) are negative extends from ω_o up to 6 (GHz), where the refractive index is expected to take a negative value. Moreover, Fig.2 shows the linear dispersion curves which is the variation of the frequency with the wave index are display the (*i*) expected reciprocal behavior, which is important in microwave signal processing technology, (*ii*) since the propagation characteristics both forwards to the right and symmetrical (k>0), (*iii*) the frequency is decreasing with increasing the wave number k and this happen only in the propagation of TE-magnetostatic surface waves. Its also notice that, by increasing the applied external magnetic field, $\mu_0 H_0$ by 0.52T, 0.55T and 0.6T respectively, the TE surface waves disappeared in the proposed region, $4 \le f \le 10$ (GHz). For f < 4(GHz), where $\mu_v > 0$ and $\varepsilon < 0$, the medium is transparentmedium and the guiding structure becomes a metallic as in Fig. 3, and for 6 < f < 10 (GHz) there is no physical solution while for $f \ge 10$ (GHz), where ($\mu_v > 0$, $\varepsilon > 0$), the physical solutions are starting to appear and the media becomes a dielectric.

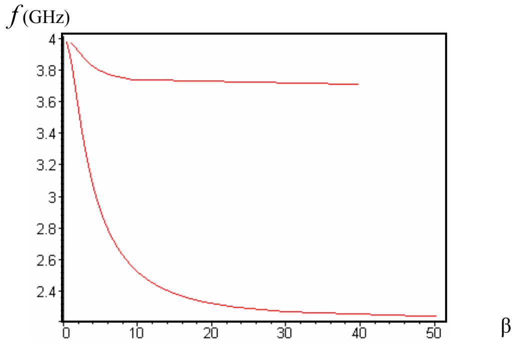

Fig. 3. Shows the Dispersion characteristics considered as a metal in case (f < 4 GHz), $\mu_0 H_0$ = 0.5 T, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ =1.76 × 10^{11}rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56

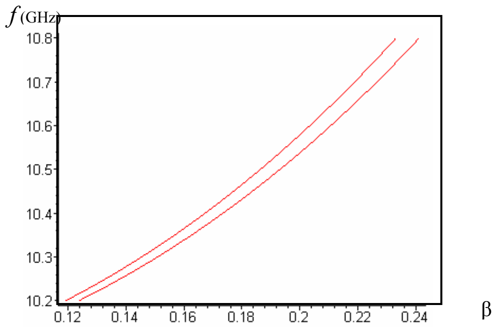

Fig. 4. Shows the Dispersion characteristics considered as a dielectric in case (f > 6 GHz), $\mu_0 H_0$ = 0.5 T, μ_B = 1.25, $\mu_0 M_0$ =0.1750 T, γ =1.76 × 10^{11} rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56

On the other hand, we increased the applied external magnetic field $\mu_0 H_0$ by 0.2T and 0.3T for $f \le 4$(GHz), fig. 5 and for $f \ge 10$(GHz), fig. 6, it is seen that the propagation in the forward direction began to decrease respectively.

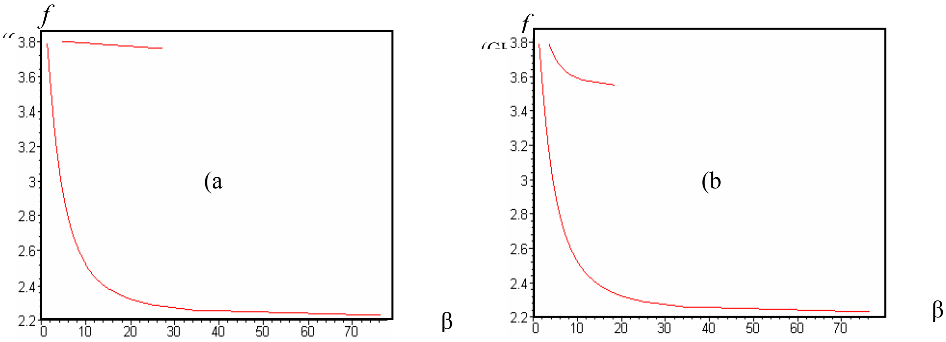

Fig.(5a-b). Shows the Dispersion characteristics in case ($f < 4$ GHz), $\mu_0 H_0$ = 0.2T, 0.3T respectively, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ =1.76 ×10^{11} rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56

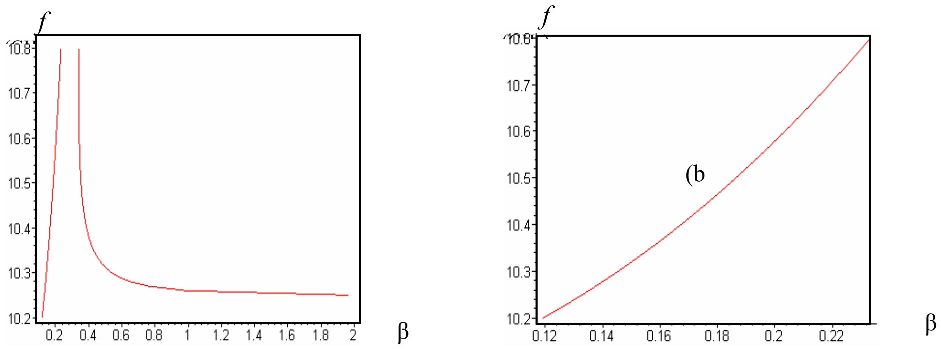

Fig.(6a-b). Shows the Dispersion characteristics in case ($f \geq 10$ GHz), $\mu_0 H_0$=0.2T, 0.3T respectively, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ =1.76 ×10^{11} rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56

CONCLUSION:

A general dispersion equation that describes the propagation of magnetostatic surface waves in a YIG film, bounded by a left handed cover, has been derived theoretically. We proposed here an approach describing a new type of reciprocal behavior in a two-layer structure, which is very promising for opto-microwave electronic devices.

REFERENCES:

1. R. A. Shelby, D. R. Smith, S. Schultz, Science 292, 77 (2001).
2. V. G. Veselago, Sov. Phys. Usp. 10, 509, (1968).
3. J. B. Pendry, S. A. Ramarkrishna, J. Phys.: Cordens. Matter 14, 6345 (2003).
4. Smith, D. R., P. Rye, D. C. Vier, A. F. Starr, J. J. Mock, T. Perram, "Design and Measurement of Anistropic Meatamaterials that Exhibit Negative Refraction", IEICE TRANS. ELECTRON., Vol. E87 – C, No. 3, (2004).
5. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat – Nasser and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity", Phys. Rev. Lett., Vol. 84, No. 18, 4184, (2000).
6. R. Ruppin, J. Phys. Condens. Matter, Vol. 13, 1811–1819, (2001).
7. Damon, R. W. and J. R. Eshbach, "Magnetostatic modes of ferromagnetic slab", J. phys. Chem.. solids, vol. 19, 308, 1961.
8. M. M. Shabat, "Nonlinear Magnetostatic Surface Waves in a gyromagnetic film" philosophical Magazine B, Vol. 73, No. 4, 669 – 676, (1996).
9. A. D. Boardman, M. Bertolott and T. Twardowski "Nonlinear Waves in Solid State Physics" Plenum Press, New York, (1990).
10. A. Hartestein, E. Burstein, A. A. Maradudin, R. Brewer and R. F. Wallis "Surface Polaritons on semi-infinite gyromagnetic media" J. phys. C: Solid state phys., Vol. 6, (1973).